Dynamic Financial Analysis as the untrodden path for company risk measurement under Solvency-II

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Dynamic Financial Analysis (DFA) is the most advance modeling process in today’s property and casualty industry-allowing us to develop financial forecasts that integrate the variability and interrelationships of critical factors affecting our results. Through the modeling of DFA, we see the company’s relevant random variables is based on the categorization of risks which is generated solvency testing where the financial position of the company is evaluated from the perspective of the customers. The central idea is to quantify in probabilistic terms whether the company will be able to meet its commitments in the future.

DFA in the capital budgeting decision process of a company launching a new invention and predicting the impact of the strategic decision on the balance sheet in a horizon of few years.

To recognize the few factors that will affect the asset liability cash flow are demand uncertainty, sales volatility, credit risk, volatility in the price of raw materials cost of capital to name a few. Each of these random variables can be stochastically simulated either based on the distribution of retrospective data or under strategic assumptions. When simulated in a combined way the future cash flows can be predicted which in return would dictate the capital requirements in the future. Depending on the capital structure of the company and simulated interest rate in the capital market the final earnings volatility of the company can be predicted to identify the return and associated risks.

But still we’re not convinced that DFA will be the paradigm to do Solvency 2, and risk-based insurance pricing, for insurance companies.

(Initial draft)

Zürich-7th Feb. 2007
Since some years Dynamic Financial Analysis (DFA) is the most advance modelling process in today’s property and casualty industry—allowing us to develop financial forecasts that integrate the variability and interrelationships of critical factors affecting our results. Our DFA models can incorporate company’s unique circumstances, such as marketplace, management philosophy and business operations, to give a customized range of what if scenarios. The DFA tools are a series of interrelated models – each geared to perform a specific type of analysis – that can be tailored and applied, either singly or in concert, to a particular business problem.

"DFA is a systematic approach based on large-scale computer simulations for the integrated financial modelling of non-life insurance and reinsurance companies aimed at assessing the risks and the benefits associated with strategic decisions.” –Paul Embrechts

We are exploring the proposal on Dynamic Financial Analysis (DFA), which is based on stochastic simulation (also called Monte Carlo simulation because it is basically the only means that allows one to deal with the long time horizons present in insurance and with the combination of models for a large number of interacting risk factors). Also herewith we should cover several questions as following:

1. Explanations about Dynamic Financial Analysis in economic and non-life insurance term, why it is important to research?
2. What kinds of works or models are used so far?
3. How academic and corporate world is using DFA for applications?
4. What do we expect from our research?
5. What are the future motivations of this research?

Areas of Applications: Business mix, Reinsurance, Asset allocation, Capital, Profitability, Solvency, Compliance, Sensitivity, Dependency.

Objectives of DFA:

DFA borrows many well-known concepts and methods from economics and statistics. It is domain of the financial management of the firm or industry. As such it is committed to management of profitability and financial stability (risk control function of DFA). While the first task aims at maximizing shareholder value, the second one serves maintaining customer value. Within these two seemingly conflicting coordinates DFA tries to facilitate and help justify or explain strategic management decisions with respect to: Strategic asset allocation, Capital allocation, Performance measurement, Market strategies, Business mix, Pricing decisions, Product design and others.

Actually DFA goes beyond designing an asset allocation strategy. In fact, portfolio managers will be affected by DFA decisions as well as with underwriters. Concrete implementation and application of a DFA model depends on two fundamental and closely related questions to be answered beforehand and three postulates:

1. Who is the primary beneficiary of a DFA analysis (shareholder, management, policyholders)?
2. What are the company individual objectives?
3. How to design of a globally applicable risk-based solvency framework for the calculation of capital requirement for non-life, life and health issues?

Solvency capital requirement = (total balance sheet requirement) - (liability requirement)
DFA is to financial planning what confidence intervals are to loss reserving.
DFA allows users to look at the distribution of potential financial developments under specific conditions.
DFA allows users to change the conditions and examine the effects of the change.

Components of modelling:

- **Underwriting Module**
  - Loss Frequency and Severity
  - Rates and Exposures
  - Underwriting Cycle
  - Jurisdictional Risk
  - Aging Phenomenon
- **Financial Module**
  - Interest Rate Generator
  - Inflation Effect
  - Investment Valuation and Performance
- **Catastrophes**
- **Taxes**
- **Reinsurance**

Research directions or questions (preliminary):

a) How DFA is modelled **Interest Rate and FX Rates** (explanations should be with general stylised facts of yield curve and FX data sets)?

b) How DFA helps Catastrophe Modelling? (Catastrophe analysis focuses on exposure, probable maximum loss and average annual loss, combined with summary overview. It also quantifies the key risk drivers in a portfolio by identifying the location and characteristics of risk. The proposition models loss results summarized by region, program or profit centre; identifies the portfolio risk characteristics driving losses; and compares client results with industry peers and the wider market. The information helps to optimise the risk portfolios to proactively manage the risk and reinsurance costs. To designing a growth template to target exposures that will maximize premium while minimizing estimated losses.)

c) DFA is the most advanced modelling process in today's property and casualty industry-allowing you to develop financial forecasts that integrate the variability and interrelationships of critical factors affecting the results. DFA models can incorporate company’s unique circumstances, such as marketplace, management philosophy and business operations, to give a customized range of what if scenarios. The DFA tools are a series of interrelated models - each geared to perform a specific type of analysis - that can be tailored and applied, either singly or in concert, to a particular business problem. Such as:

- Macroeconomic model: Includes GDP growth, inflation, interest rates, stock returns, dividend yields, and bond default rates, and serves as a basis for generating asset and liability projections in literally thousands of simulated environments.
- Business portfolio optimiser: Enables you to view your entire company - including both assets and lines of business - as a single portfolio, and provides guidance in selecting business mix strategies.
- Underwriting scenario generator: Under varying economic scenarios, uses assumptions regarding premium, losses, and expenses on gross and ceded bases to create projections of cash flows and reserves.
• Investment model: Aggregates cash from all sources, and projects investment returns in line with your investment strategy and risk preferences, arriving at a projection of year-end assets.

e) Comparison with DFA Asset Liability Management and Cash Flow at Risk (with example).
f) For non-life insurance company what is the cost of capital spread should be promised to the shareholder? (It’s the connection to relate DFA via CAPM-model)
g) Full balance sheet approach: 1. Estimate liabilities (Stock Model), 2. Dependence structure between asset and liability.
h) What is the minimal amount for the initial balance $B(0)$ such that the deal is acceptable for the reinsurer?

Reuse of models:

In the DFA we reuse of already-available models for certain risk factors which is not to problematic for claims models (modulo dependence) but it can be a problem on the financial and economic side. We also models DFA from economic and econometrics point of view which often at explaining the usual behavior of risk factors and it also often fails to reproduce unusual features, although these are the most dangerous parts of analysis for a risk manager. There are another core of models from mathematical finance which is the structure often governed by mathematical convenience, i.e. closed form solutions for option prices. For DFA, the faithful reproduction of the statistical behavior of the real-world risks is of utmost importance.

Company & strategy model reflects the internal financial structure of the company. That means consolidation of various lines of business, investment portfolio and reinsurance. It Models the company’s reaction on the development of risk factors and often very detailed: creates projections of company’s balance sheet. The Model contains parameters that are under management’s control, e.g. reinsurance structures and retentions, investment portfolio weights. A set of values for these parameters corresponds to a strategy. The goal of a DFA study also compare different possible strategies by Kaufmann et al., 2001; Hodes et al., 1999; Dynamo; Daykin et al., 1994, CAS, 1999.

Strategy modeling: Current DFA fails to incorporate properly managerial flexibility. Output of DFA simulations is often questionable because it is based on the implicit assumption that management is unable to react in case of adverse developments over time. DFA often models strategies (i.e. parameters under the control of management) in a non-adaptive way, i.e. as fixed values. However, DFA models several time periods, and each of these time periods is usually substantially long, i.e. one year. Given all these factors, it is not realistic to assume that management will not adapt a strategy if risk factors develop dramatically (upside or downside) in a particular scenario. The portfolio weight of equity shares in case a scenario displays a dramatically decline in the equity market.

Output & evaluation: The Output always equals to the values resulting from the application of the company model and parameterized with some strategy on the risk factor scenarios generated by scenario generator. The we can go up to full pro-forma balance sheets, e.g. in Dynamo. The Monte Carlo simulation depicts the large number of result scenarios, which analyze empirical P&L distribution, compute measures of risk and return, see e.g. Artzner et al., 1999. The risk nothing but the return analysis across several strategies, then we do NPV
computations, sensitivity analysis, efficient frontier and the healthy comparison between total risk vs. systematic/non-systematic risk, see Cumberworth et al., 1999.

**Analyzing DFA Results Through Efficient Frontiers:**

The common sketch of DFA is the *efficient frontier* concept widely used in modern portfolio theory (see Markowitz). A company has to prefer a return measure (e.g. expected surplus) and a risk measure (e.g. expected policyholder deficit, see Lowe and Stanard, or worst conditional mean as a coherent risk measure, see Artzner, Delbaen, Eber and Heath). A strategy is called *efficient* if there is no other one with lower risk at the same level of return, or higher return at the same level of risk. For each level of risk there is a maximal return that cannot be exceeded, giving rise to an efficient frontier. There is no absolute certainty whether a strategy is really efficient or not. DFA is not necessarily a method to come up with an optimal strategy. DFA is predominantly a tool to compare different strategies in terms of risk and return. Though efficient frontiers are a good means of communicating the results of DFA because they are well-known, some words of criticism are in place. Cumberworth, Hitchcox, McConnell and Smith have pointed out that there are pitfalls related to efficient frontiers one has to be aware of. They criticize that typical efficient frontier uses risk measures that mix together systematic risk (non-diversifiable by shareholders) and non-systematic risk, which blurs the shareholder value perspective.

**Solvency Testing:** The perception closely connected to DFA is solvency testing where the financial position of the company is evaluated from the perspective of the customers. The inner idea is to count in probabilistic terms whether the company will be able to meet its promises in the future. This interprets into determining the necessary amount of capital given the level of risk the company is exposed to. DFA serves as a solvency testing tool as well. More information about solvency testing can be found in Schnieper.

**Structure of a DFA Model:** Nearly all DFA models consist of three major parts. The *stochastic scenario generator* produces realizations of random variables representing the most important drivers of business results. A realization of a random variable in the course of simulation corresponds to fixing a scenario. The second data source consists of company specific *input* (e.g. mean severity of losses per line of business and per accident year), assumptions regarding model parameters (e.g. long-term mean rate in a mean reverting interest rate model), and strategic assumptions (e.g. investment strategy). The last part, the *output* provided by the DFA model, can then be analyzed by management in order to improve the strategy, i.e. make new strategic assumptions. This can be repeated until management is convinced by the superiority of a certain strategy. As pointed out in Cumberworth, Hitchcox, McConnell and Smith [10] interpretation of the output is an often neglected and non-appreciated part in DFA modelling. For example, an efficient frontier leaves us still with a variety of equally desirable strategies. At the end of the day management has to decide for only one of them and selection of a strategy based on preference or utility functions does not seem to provide a practical solution in every case.

**Stochastically Modeled Variables:**

To build an appropriate model firstly we need to identify the key random variables affecting asset and liability cash flows. Afterwards it has to be decided whether and how to model each or only some of these factors and the relationships between them. This decision is influenced by considerations of a trade-off between improvements of accuracy versus increase in complexity, which is often felt being equivalent to a reduction of transparency.
The risks affecting the financial position of a non-life insurer can be categorized in various ways. For example, pure asset, pure liability and asset/liability risks. We believe that a DFA model should at least address the pricing or underwriting risk (risk of inadequate premiums), reserving risk (risk of insufficient reserves), investment risk (volatile investment returns and capital gains) and catastrophes.

![Figure 2: Main structure of a DFA model.](image)

We could have also mentioned credit risk related to reinsurer default, currency risk and some more. DFA discussion of the possible impact of exchange rates on reinsurance contracts, see Blum, Dacorogna, Embrechts, Neghaiwi and Niggl. A critical part of a DFA model is the interdependencies between different risk categories, in particular between risks associated with the asset side and those belonging to liabilities.

In the implementation part we assumed the interest rates were strongly correlated with inflation, which itself influenced future changes in claim size and claim frequency.

We explicitly considered on the liability side with four sources of randomness: non-catastrophe losses, catastrophe losses, underwriting cycles, and payment patterns. We simulated catastrophes separately due to quite different statistical behavior of catastrophe and non-catastrophe losses. In general the volume of empirical data for non-catastrophe losses is much bigger than for catastrophe losses. Separating the two led to more homogeneous data for non-catastrophe losses, which made fitting the data by well-known (right skewed) distributions easier. Also, our model implementation allowed for evaluating reinsurance programs. Testing different deductibles or limits is only possible if the model is able to generate sufficiently large individual losses, see Kauffman et al 2001. We also have the experience a rapid development of a theory of distributions for extremal events (see Embrechts, Klüppelberg and Mikosch, and McNeil ). Therefore, we considered the separate modeling of catastrophe and non-catastrophe losses as most appropriate. For each of these two groups the number and the severity of claims were modeled separately. Another approach would have been to integrate the two kinds of losses by using heavy-tailed claim size distributions.

Losses are not only characterized by their (ultimate) size but also by their piecewise payment over time. This property increases the uncertainties of the claims process by introducing the time value of money and future inflation considerations. As a consequence, it is necessary not only to model claim frequency and severity but the uncertainties involved in the settlement process as well. In order to allow for reserving risk we used stochastic payment patterns as a means of estimating loss reserves on a gross and on a net basis.
In the proposal we pointed out that our intention was to present a DFA model framework in several perspective in terms of business decisions and goals.

**Interest Rates:** By Daykin, Pentikäinen and Pesonen we suppose strong correlation between general inflation and interest rates. The primary stochastic driver is the (instantaneous) short-term interest rate. This variable determines bond returns across all maturities as well as general inflation and superimposed inflation by line of business. An alternative to the modeling of interest and inflation rates as outlined in our perspective research and probably well-known to actuaries is the *Wilkie model*, see Wilkie, and Daykin, Pentikäinen and Pesonen.

Short-Term Interest Rate: There are many several interest rate models used by financial economists. The surveys of interest rate models have grown considerably in the corporate finance. The following references represent an arbitrary selection: Ahlgrim, D’Arcy and Gorvett, Musiela and Rutkowski and Björk. The general features of interest rate movements can be taken from Ahlgrim, D’Arcy and Gorvett with specific cases like: 1. Volatility of yields at different maturities varies, 2. Interest rates are mean-reverting, 3. Rates at different maturities are positively correlated, 4. Interest rates should not be allowed to become negative, 5. The volatility of interest rates should be proportional to the level of the rate.

The practicality of the modeling structures also raised by Rogers. According to Rogers an interest rate model should be flexible enough to cover most situations arising in practice, simple enough that one can compute answers in reasonable time, well-specified, in that required inputs can be observed or estimated, realistic, in that the model will not do silly things.

We determined to express through the one-factor Cox–Ingersoll–Ross model. Which belongs to the class of equilibrium based models where the instantaneous rate is modeled as a special case of an Ornstein–Uhlenbeck process:

\[
dr_t = -\theta(r_t - \mu)dt + \sigma dW_t,
\]

where \(\theta\), \(\mu\) and \(\sigma\) are parameters. This equation is solved by using a variation of parameters argument.

If you analyze the Ornstein–Uhlenbeck process applying Itô’s lemma to the function \(f(r_t, t) = r_t e^{\theta t}\) to get

\[
df(r_t, t) = \theta r_t e^{\theta t} dt + e^{\theta t} \sigma dW_t.
\]

The Ornstein-Uhlenbeck process (an example of a Gaussian process that has a bounded variance) admits a stationary probability distribution, in contrast to the Wiener process. Which is a continuous-time Gaussian stochastic process with independent increments used in modelling Brownian motion and some random phenomena observed in finance. It is one of the best-known Lévy processes. For each positive number \(t\), denote the value of the process at time \(t\) by \(W_t\).

Moreover, in risk management framework, however, one is more interested in the potential downside of the target variable. A very popular measure for downside risk is the **Value-at-Risk (VaR)**, which is simply the \(p\)-quantile for the distribution of \(Y\) for some probability \(0 < p < 1\). It is easily computed as
\[ \text{VaR}_p(Y) = \min \left\{ y_{(k)} : \frac{k}{N} > p \right\}, \] Where \( y_{(k)} \) is the \( k \)th order of \( y_1, \ldots, y_N \).

for example, expected policyholder deficit, twisted means or Wang and Esscher transforms. Another downside risk measure, extending the already introduced VaR, is the Tail-VaR, defined as contrary to most other risk measures including VaR and standard deviation – it belongs to the class of Coherent Risk Measure.

Stock Returns. The terms of stock option and restricted stock plans, and the flexibility afforded the board of directors in negotiating with managers, vary systematically with the characteristics of the assets being managed.

The underwriting parts of profit distributions, operating profit distributions reflecting investment as well as underwriting risk are produced by the model, such as those shown on the following figure. These can be used to translate underwriting risk into operating profit terms, or to test the effect of introducing various levels of asset risk via changes to the mix of investments.

And the asset classes of a non-life insurance company comprise fixed income type assets, stocks and real estate. Here, we confine ourselves to a description of the model employed for stocks. Modeling stocks can start either with concentrating on stock prices or stock returns (although both methods should turn out to be equivalent in the end). We followed the last approach since we could rely on a well established theory relating stock returns and the risk-free interest rate: the Capital Asset Pricing Model (CAPM), which depicts the expected return of a security or a portfolio equals the rate on a risk-free security plus a risk premium. If this expected return does not meet or beat the required return, then the investment should not be undertaken. The security market line plots the results of the CAPM for all different risks (e.g: betas). Departing back to Sharpe–Lintner, which includes: 1. The intercept is zero, 2. Beta completely captures the cross sectional variation of expected excess returns, 3. The market risk premium is positive, see for example Ingersoll. Moreover, we would like to highlight that the method of modeling stock returns represents is one of the approaches from many possibilities.

Non-Catastrophe Losses: Non-catastrophe losses of various lines of business develop quite differently compared to catastrophe losses. Traditional ratemaking techniques developed a catastrophe provision by using historical ratios of catastrophe losses to non-catastrophe losses. More recently we have seen the catastrophe provision calculated by comparing catastrophe losses to amount of insurance years. The methods are more responsive to one aspect of a changing exposure base (i.e. total amount of insurance). However, neither of these methods can properly capture the expected loss of the hurricane peril. The aging phenomenon, believed to occur for all lines of property-liability insurance, although little published information confirms this belief and to incorporate the aging phenomenon into a pricing model. The aging phenomenon describes the fact that the loss ratio – i.e. the ratio of (estimated) total loss divided by earned premiums – decreases when the age of policy increases as proposed by D’Arcy, Gorvett, Herbers, Hettinger, Lehmann and Miller.

Catastrophes: Losses through catastrophic events like windstorm, flood, hurricane, earthquake, etc. we will configure with integrated non-catastrophic and catastrophic losses by using heavy-tailed distributions, see Embrechts, Klüppelberg and Mikosch. Nevertheless, we decided for separate modeling.

Underwriting Cycles: The cycles can vary significantly between countries, markets and lines of business and sometimes their appearances are veiled by smoothing results. Which includes the time lag effect of the pricing procedure, trends, cycles and short-term variations of claims, fluctuations in interest rate and market values of assets. We shall also work on this approach.
Using homogeneous Markov chain model (in discrete time) similar to D’Arcy, Gorvett, Hettinger and Walling.

**Payment Patterns:** Besides claim numbers and severities issue we need to explain in a more dedicated way to explaining how we managed to model the uncertainties of the claim settlement process, i.e. the random time to payment.

The loss of development factor is defined as:

\[ d_{t_1,t_2} := \frac{\sum_{t=0}^{t_1-1} Z_{t_1,t}}{Z_{t_1,t_2}}, t_2 \geq 1 \]

We configure a chain-ladder type procedure (for the chain-ladder method, see Mack), where 

\[ d_{t_1,t_2} = \log \text{normal} \left( \mu_{t_2}, \sigma_{t_2}^2 \right); \quad \mu_{t_2} = \text{estimated logarithmic loss development factor for development year } t_2, \text{ based on historical data} \]

\[ \sigma_{t_2}^2 = \text{estimated logarithmic standard deviation of loss development factors, based on historical data} \]

Several important references on stochastic models in loss reserving are generated in Christofides and Taylor.

**Option Pricing aspect:** A new perspective also has been arrived to measure option pricing both in discrete and continuous time in the framework of Markov and Semi-Markov processes proposed by Janssen, 1995 and we should compare the model with our results.

**Scenario generation: problems:**

In general: high numbers of dependent risk factors; i.e. dependence modeling is an important issue. The structural models, e.g. the Wilkie model, see Daykin et al., 1994, alternatively: statistical dependence models. Regarding statistical dependence models (see Embrechts et al., 1999) with linear correlation is often an inappropriate model for dependence, particularly in case of non-elliptical distributions and a possible alternative: copula methodology also impressive indeed. We also found the important modeling extremal events (see Embrechts et al., 1997) which includes often not appropriately reflected by stoch Models that rely on the Gaussian distribution (“heavy-tailed risks”) that embarks particularly difficulties, multivariate extremes also. We could analyze another big issue herewith, means low amounts of data available for the calibration of the models implies high model and parameter uncertainty.
Modelling: First of all we should adapt the continuation of modelling structure with the general class of price kernel models (state price deflator models). The main motivation is that with this approach, from the requirement point of view we need to fix-up and which simultaneously incorporate the yield curves of currency rates. We optimize the expected discounted utility of future consumption as following (see Blum, 2005):

$$\max_c E \left[ \int_0^\infty e^{-r_s} \theta(c_s) ds \bigg| F_t \right].$$

where $\theta(.)$ is the von Neumann-Mogenstern utility function of the representative agent, $c=(c_t)_{t \geq 0}$ is consumption stream and $\rho$ is constant.

For valuation point of view the price kernel is assigning fair present values to some future, uncertain cash flows which depend on optimal stopping time (e.g American options) and also on some exogenous random variable independent of $F_t$. Modern actuarial valuation techniques have with peer motivations to deal with price kernel like the Esscher or Wang transforms which is also practicable if all systematic (hedgeable) risk has been removed.

In the continuation of modelling amalgamation under price kernel the ultimate decision is to select the exponential quadratic function accordingly in conjunction with the multivariate Ornstein-Uhlenbeck process. Then in the calibration of the models section we use Generalized Methods of Moment (GMM) as a tool for calibrating the interest rate and FX model to historical data.

*** Nonparametric Estimation with Kernel and some empirical findings with Non-linear Factor Analysis:

(This section would provide after the selection of this first and incomplete draft!!!)
**Example:** The coupon rate is set equal to the market interest rate at the duration of the payout pattern. Then we are experienced to measure the valuation and embedded options in terms of portfolio of securities from the complete market spanned by interest rate and FX rate model. The Reinsurer can also hedge the exchange rate risk by using derivatives.

**Figure:** Value-at-Risk (VaR, left) and Expected Shortfall (ES, right) to estimate the influence of currency fluctuations. The ES is a more conservative and more realistic estimate. Assuming it goes wrong, it tells you how wrong it will go. The VaR gives an estimate on the the border of where it will be in 99% or 95% of the cases. Portfolios consisting of several currencies, in addition to ensure payment dates create interesting challenges.

**DFA and Risk Management:** DFA takes a sensible point of view regarding integrated, holistic view of the company, no artificial separation of aspects that belong together, time axis of business is taken into account; And most importantly DFA is practically feasible takes the investors’ perspective.

High model risk through joint modeling of many risk factors that means company models focus on imitation of accounting rather that on economic investigation of value creation. Which are too detailed company models bear the risk of incorporating things that cannot be reasonably modeled (e.g. accounting details). The managerial flexibility is often not incorporated in the models. And it could be the analysis of results is often too superficial; so we need alternative evaluation methods could extract more information of the simulation results.

Much work still needs to be done in order to make DFA a fully reliable analysis and decision-making tool for all situations.

**Acknowledgement:** Most of the parts of the paper have been influences pearly and taken several things as an example and from seminal papers of EMBRECHTS P., MCNEIL A, KAUFMANN R., BLUM P, DACOROGNA M. and all.

**Bibliography:**


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